

- VLADIMIR KANOVEI, *On sets that hereditarily belong to countable OD sets.*  
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By Tzouvaras [4] (see also [1, 2]), the class **NT** of all sets *nontypical in the Russell sense* consists of all sets  $x$  which belong to countable ordinal-definable (OD) sets  $X$ . Fuchs [1] called such sets *blurry definable*. As usual, a set  $x$  is *hereditarily nontypical*, in symbol  $x \in \mathbf{HNT}$ , if the transitive closure satisfies  $\text{TC}(x) \subseteq \mathbf{NT}$ .

The class **HNT** is transitive, satisfies  $\mathbf{HOD} \subseteq \mathbf{HNT}$ , and by [4] is a model of **ZF** (not necessarily of **ZFC**). The axiom of constructibility obviously implies  $\mathbf{HOD} = \mathbf{HNT} = \mathbf{V}$  (the universe). Tzouvaras [4] asks whether the other possible combinations

- (I)  $\mathbf{HOD} = \mathbf{HNT} \subsetneq \mathbf{V}$   
 (II)  $\mathbf{HOD} \subsetneq \mathbf{HNT} = \mathbf{V}$   
 (III)  $\mathbf{HOD} \subsetneq \mathbf{HNT} \subsetneq \mathbf{V}$

are consistent. We answer this in the affirmative.

**THEOREM 1.** *Each of (I), (II), (III) is consistent with the theory **ZFC** + “**HNT** is a model of **ZFC**”.*

A model for (I) can be defined by adjoining a Cohen-generic real  $a$  to  $\mathbf{L}$ . In such an extension we have  $\mathbf{L} = \mathbf{HOD} = \mathbf{HNT} \subsetneq \mathbf{V} = \mathbf{L}[a]$ .

A model for (II) can be defined by adjoining a generic real  $b$  to  $\mathbf{L}$ , such that the  $\mathbf{E}_0$ -equivalence class  $[b]_{\mathbf{E}_0}$  is a (countable) lightface  $\Pi_2^1$  set containing no OD elements as in [3]. In such an extension we have  $\mathbf{L} = \mathbf{HOD} \subsetneq \mathbf{HNT} = \mathbf{V} = \mathbf{L}[b]$ .

A model for (III) can be defined by adjoining a generic pair of reals  $a, b$  to  $\mathbf{L}$ , where  $a, b$  are as above. In such an extension we have

$$\mathbf{L} = \mathbf{HOD} \subsetneq \mathbf{HNT} = \mathbf{L}[b] \subsetneq \mathbf{V} = \mathbf{L}[a, b].$$

See [2] for the details of this model.

[1] G. FUCHS, *Blurry definability*, *Mathematics*, vol. 10, no. 3 (2022), paper no. 452.

[2] V. KANOVEI, V. LYUBETSKY, *A generic model in which the Russell-nontypical sets satisfy ZFC strictly between HOD and the universe*, *Mathematics*, vol. 10, no. 3 (2022), paper no. 491.

[3] V. KANOVEI, V. LYUBETSKY, *A definable  $\mathbf{E}_0$  class containing no definable elements*, *Archive for Mathematical Logic*, vol. 54 (2015), pp. 711–723.

[4] A. TZOUVARAS, *Typicality á la Russell in Set Theory*, *Notre Dame Journal of Formal Logic*, vol. 63, no 2 (2022), pp. 185-196