Silver measurability and more in the Laver model

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Abstract : This presentation is the result of a joint work with Michel Gaspar and in this we are going to delve into the well-known research field of studying different kinds of measurabilities (the most well-known of them being *Lebesgue* and *Baire* measurabilities) calibrated against the projective hierarchy of the sets reals, with a focus on proving that *Laver* measurability for all $\mathbf{\Delta^1_2}$ sets can be obtained without obtaining the *Silver* measurability of all $\mathbf{\Delta^1_2}$ sets as a byproduct, i.e, there will be atleast one $\mathbf{\Delta^1_2}$ set which is not Silver measurable. In other words, $\mathbf{\Delta^1_2}(\mathbb{L}) \nRightarrow \mathbf{\Delta^1_2}(\mathbb{V})$ is consistent. This solves the question listed as [2, Open question 6.3]. It has also been mentioned open in the works of Laguzzi, Brendle, Ikegami and Loewe.

The other direction (i.e, the consistency of $\mathbf{\Delta^1_2}(\mathbb{V}) \nRightarrow \mathbf{\Delta^1_2}(\mathbb{L})$) is quite well-known.

Apart from that we also show the consistency of $\mathbf{\Delta^1_2}(\mathbb{L}) \nRightarrow \mathbf{\Delta^1_2}(E_0)$. We also shall prove that the existence of a splitting real over $\mathbf{L}[r]$ for every real r does not imply $\mathbf{\Delta^1_2}(\mathbb{V})$. The former is an open question listed as [3, question 5.2] by Geschke and Gaspar in their ongoing work about Silver regularity and the latter is an open question listed as [1, question 2] in the works of Brendle, Halbeisen and Loewe.

References

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