λ_2-thick graphs and their applications

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It is well known that several methods of set theory are very useful in various areas of mathematics, especially in modern measure theory. Set-theoretical aspects of the existence of a Lebesgue nonmeasurable subset of R and the measure extension problem plays an important role in measure theory. Let us remark that the existence of nonmeasurable sets trivially implies the existence of nonmeasurable real-valued functions in the Lebesgue sense. (see [2],[3])

A subset X of R^2 is λ_2-thick (or λ_2-massive) in the Euclidean plane R^2 if, for every λ_2-measurable set $Z \subset R^2$ with $\lambda_2(Z) > 0$, the relation $X \cap Z \neq \emptyset$ holds true, where λ_2 is the standard two-dimensional Lebesgue measure on the Euclidean plane.

Sierpinski showed that there are injective functions acting from R into R whose graphs are λ_2-thick subsets of the plane R^2.

Noticed that if a subset X of R^2 is λ_2-measurable and λ_2-thick simultaneously, then it is of full λ_2-measure, i.e., $\lambda_2(R^2 \setminus X) = 0$. If the set X in the plane is not of full λ_2-measure but is λ_2-thick, then X is not λ_2-measurable.(see.[1],[4])

In the present talk we discuss, λ_2-thick subsets of the plane R^2, functions acting from R into R whose graphs are λ_2-thick and set-theoretical aspects of their applications in the study of invariant (quasi-invariant) measure extension problem. Also, we will demonstrate several applications of the method for obtaining nonseparable R-invariant extensions of λ based on certain properties of group homomorphisms which have thick graphs.

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