DYNAMICS AND THE CLOPEN TYPE SEMIGROUP: SOME REMARKS AND MANY QUESTIONS

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Connsider an action α of a countable group Γ a compact, 0-dimensional, Hausdorff space X (we are mostly interested in the case where X is the Cantor space, though X is allowed to be non-metrizable, for instance it could be the Stone-Cečh compactification $\beta\Gamma$ of Γ). The *topological full group* $[[\alpha]]$ of this action is the set of all $g \in \mathrm{Homeo}(X)$ such that there exists a clopen partition $A_1, \ldots, A_n$ of X and elements $\gamma_1, \ldots, \gamma_n$ of Γ such that for all $x \in A_i$ one has $g(x) = \gamma_i(x)$.

The best-understood case if that of minimal $\mathbf{Z}$-actions on the Cantor space, where topological full groups play an important role,. Such actions have the *dynamical comparison property*: whenever A, B are clopen and such that $\mu(A) < \mu(B)$ for every invariant Borel probability measure, there exists an element of $[[\alpha]]$ mapping A into B. Also of interest is the existence of a dense locally finite group of $[[\alpha]]$, as well as the question of whether the action is *saturated*, meaning that as soon as $\mu(A) = \mu(B)$ for every invariant Borel probability measure there exists $g \in [[\alpha]]$ such that $g(A) = B$.

In recent years, dynamical comparison has been intensively studied, in large part for its relation to C^*-algebraic problems, and a connection between dynamical comparison and strict unperforation of the clopen type semigroup (an object related to equidecomposability, originating in work of Tarski on amenability) has been noted by several authors. We will see that saturation of the action and (at least for minimal actions on the Cantor space) existence of a dense locally finite subgroup can also be seen as properties of the clopen type semigroup. I intend to briefly discuss the case of the Stone-Cečh compactification and universal minimal flow of Γ, then some properties of generic minimal actions of amenable groups on the Cantor space and (assuming time allows) mention an embarrassing number of open problems which I find appealing.

The talk will be a survey aimed at a broad audience; I will assume no familiarity with topological dynamics or type semigroups.

This talk is based on joint work with S. Robert (Université Lyon 1).