If we replace first order logic by second order logic in the original definition of Gödel's inner model L, we obtain HOD. In this talk we consider inner models that arise if we replace first order logic by a logic that has some, but not all, of the strength of second order logic. Typical examples are the extensions of first order logic by generalized quantifiers, such as the Magidor-Malitz quantifier, the cofinality quantifier, or stationary logic. It can be shown that both L and HOD manifest some amount of formalism freeness in the sense that they are not very sensitive to the choice of the underlying logic.

On the other hand, the cofinality quantifier gives rise to a new robust inner model between L and HOD. Assuming a proper class of Woodin cardinals the regular cardinals above aleph-1 of V are weakly compact in the inner model arising from the cofinality quantifier and the theory of that model is (set) forcing absolute and independent of the cofinality in question. Assuming three Woodin cardinals and a measurable above them, if the construction is relativized to a real, then on a cone of reals the Continuum Hypothesis is true in the relativized model.

A potentially bigger inner model C(aa) arises from stationary logic. Assuming a proper class of Woodin cardinals, or alternatively MM-plus-plus, the regular uncountable cardinals of V are measurable in the inner model C(aa), the theory of C(aa) is (set) forcing absolute, and C(aa) satisfies CH. We introduce an auxiliary concept that we call club determinacy, which simplifies the construction of C(aa) greatly but may have also independent interest. Based on club determinacy, we introduce the concept of aa-mouse which we use to prove CH and other properties of the inner model C(aa). Finally, we discuss a delicate matter related to the Axiom of Choice in the inner model C(aa) and in inner models of the same kind.

This is joint work with Juliette Kennedy and Menachem Magidor.