

UNIVERSAL GRAPHS BETWEEN A STRONG LIMIT SINGULAR AND ITS POWER

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While a classical result implies that assuming $2^\kappa = \kappa^+$ there exists a universal graph of cardinality κ^+ , and it is not difficult to argue that there are no universal graphs after adding enough Cohen subsets [KS92]. Under non-trivial cardinal arithmetic constraints it is much more difficult to prove (consistency results on) existence of universal graphs. By an old result of S. Shelah, consistently $2^{\aleph_0} > \aleph_1$ and there exists a universal graph of cardinality \aleph_1 [She90] (and a similar result holds with regular cardinals $\kappa < \lambda$ instead of \aleph_0, \aleph_1).

At successors of strong limit singulars only weak universals were known to be consistent: M. Džamonja and S. Shelah proved that consistently (assuming a supercompact cardinal), there exists a strong limit singular μ with $\mu^{++} < 2^\mu$ such that there is a graph of cardinality μ^{++} which is universal for graphs of cardinality μ^+ [DS03]. Later J. Cummings, M. Džamonja, M. Magidor, C. Morgan, S. Shelah proved this for arbitrary cofinality in [CDM⁺17].

Improving both latter results, we showed that consistently (relative to a supercompact) for some strong limit singular μ with $\mu^+ < 2^\mu$ there exists a universal graph on μ^+ . In fact the following is true [PS]:

Theorem 1. (*M.P.-S. Shelah*) *Assume κ is supercompact, **GCH** holds, $\lambda > \kappa$ is regular, $\lambda < cf(\chi)$, and $\sigma < \kappa$ is regular.*

Then for some forcing extension (preserving cardinals $\geq \kappa$ and preserving cofinalities $> \kappa$ and $\leq \sigma$) we have $2^\kappa = \chi$, κ is a strong limit singular of cofinality σ and there is a universal graph of cardinality λ .

This is an ongoing project, joint with Saharon Shelah.

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