MULTIPLICATIVE FINITE EMBEDDABILITY VS DIVISIBILITY OF ULTRAFILTERS

We continue the exploration of various aspects of divisibility of ultrafilters, adding one more relation to the picture: multiplicative finite embeddability. It is a specimen of a family of relations considered by Luperi Baglini, with the additive version first introduced by Di Nasso and Blass. The reason for observing it alongside divisibility relations is that it lies between two of them, $|_M$ and $\widetilde{|}$. The set of its minimal elements proves to be very rich, and the $\widetilde{|}$-hierarchy is used to get a better intuition of this richness. We find the place of the set of $\widetilde{|}$-maximal ultrafilters among some known families of ultrafilters (some of which are connected to finite embeddability). Finally, we introduce some new notions of largeness of subsets of $\mathbb{N}$, and compare it to other such notions, important for infinite combinatorics and topological dynamics.

References

[1] Blass A., Di Nasso M., Finite embeddability of sets and ultrafilters. Bulletin Polish Acad. Sci. Math. 63 (2015), 195-206.

[2] Luperi Baglini L., $\mathcal{F}$-finite embeddability of sets and ultrafilters. Arch. Math. Logic 55 (2016), 705–734.

[3] Šobot B., Multiplicative finite embeddability vs. divisibility of ultrafilters. Arch. Math. Logic 61 (2022), No.3–4, 535–553.